Graph Theory and its Applications

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October 17, 2012
What is a Graph?

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- Technically, a set of vertices (objects) and a set of edges (relationships between objects)
- Can be used for anything- tons of real world applications!
Examples of Graphs

Figure: The “House” graph

Figure: The caterpillar with code (2,0,1,0,1,0,1,0,2)
What is Graph Theory?

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- Graph theory is an important modeling tool used to study network designs, communication, structural design, computer science, and many other things.
- In order to efficiently study these such applications, we must be able to describe graph parameters by providing limiting cases, like bounds, because many graph parameters are difficult to compute exactly.
First Formal Application of Graph Theory

Leonhard Euler composed a paper in 1736 titled *The Seven Bridges of Königsberg*. This paper contains the first real-world application/use of graph theory!
Visualizing the Bridges of Königsberg
Other Early Applications of Graph Theory

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- 1859- William Rowan Hamilton creates a toy involving finding a path through all cities on a map- he sells his design to a toy maker, but it was never a big hit :(
Modern Applications of Graph Theory

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- Computer network security
- Cell phone tower/network selection
Scheduling Problems

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- Taking the chromatic number of this graph gives the minimum time required to finish all jobs without conflict.
Scheduling Problems

Here’s an example:
Suppose we want to schedule some final exams for computer science courses with the following course numbers:
1007, 3137, 3157, 3203, 3261, 4115, 4118, 4156.
Suppose there are no students taking both 1007 and any other class, and no students sharing the following pairs of classes:

- 3137 and 3157
- 3137 and 4118
- 3137 and 3261
- 3137 and 4115
- 3137 and 4156
- 3203 and 3261
- 3157 and 4156
- 3203 and 4115
- 3261 and 4115
- 3203 and 4115
Scheduling Problems

Courses become vertices, and two vertices are connected with an edge if the courses have a student in common:

(as it turns out you need three exam slots!)
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This is very much like a committee choice- you want to pick a committee such that either you are on the committee or you know someone who is... representation!
Other Applications of Domination

- Transportation networks
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- Distribution logistics
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- Alarm location in facilities
Other Applications of Domination

- Transportation networks
- Distribution logistics
- Alarm location in facilities
- Radio/TV/Communication tower location (with military applications)
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- We have investigated "hitting sets", or sets of vertices that cover all edges, as the number of vertices goes to infinity
- We obtained a two point concentration: see http://arxiv.org/abs/1201.5097 for more details
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Honors thesis, will appear at the Boland Undergraduate Research Symposium in the Spring, and at the Joint Meetings in Mathematics in San Diego, California, January 2013.
Subgraphs and Supergraphs

Here, we are given a random graph with certain properties
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Here, we are given a random graph with certain properties. We want to know: What “supergraphs" can be built randomly from the given subgraphs that retain the properties of the original subgraph? Applications in disease spreading and transmission, and some network design.
- Department of Mathematics and Statistics discrete mathematics seminars
- Graph theory class, offered in the spring
- Wikipedia is pretty accurate, actually!
- Open conjectures in graph theory (especially the graph reconstruction conjecture)
Thanks for attending, please feel free to email me any time at deeringj@goldmail.etsu.edu!